$$T=t+t_1=\sqrt{\frac{r}{g}}\left(\tfrac{1}{6}\pi+2\sqrt{2}-\sqrt{3}+2\log\frac{2+\sqrt{2}}{1+\sqrt{3}}\right)=1666.673 \text{ seconds}=27$$
minutes, 46.673 seconds.

$v_3=44869.668$ feet per second$=8.498$ miles$=8\frac{1}{2}$ miles per second, nearly.

When $v_1=r\sqrt{(2g/a)}$, $v_2=\sqrt{(2gr)}$, and the velocity of arriving at the surface is independent of a, the distance from the center.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

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163. Proposed by PROFESSOR R. D. CARMICHAEL, Anniston, Ala.

Prove that the equation $y^n=mx+1$ always has at least one positive integer solution (different from $y=1$, $x=0$), whatever integer values m and n may have.

Solution by S. LEFSEHETZ, Pittsburg, Pa.

The following solution is evident:

$$y=(m+1)^n,\quad x=\frac{(m+1)^n-1}{m}.$$

To find all solutions, we remark that it is enough to find all values of y such that $y^n\equiv1 \pmod{m}$. Let f be a divisor of $\phi(m)$, a number to which f appertains. Then $a^f\equiv1 \pmod{m}$. If also $a^n\equiv1 \pmod{m}$, we must have $n\equiv0 \pmod{f}$. Hence, f is a divisor of $dv[n, \phi(m)]$. Therefore we take the $\phi(m)$ numbers smaller than m and prime to it, we form the exponents to which they appertain and keep them if their exponents divide $dv[n, \phi(m)]$. If x be such a value, $y=x+km$ is a solution, the corresponding value of x being $\frac{y^n-1}{m}$, which is integral since $y^n\equiv1 \pmod{m}$.

AVERAGE AND PROBABILITY.

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200. Proposed by PROFESSOR R. D. CARMICHAEL, Anniston, Ala.

A line $AB=l$ is extended to P making $BP=p$. If a point D is taken at random in BP, what is the mean value of $AD.DP$?

Solution by J. EDWARD SANDERS, Weather Bureau, Chicago, Ill.

Let $x=BD$. Then $AD=l+x$, $DP=p-x$, and $AD.DP=(l+x)(p-x)$.

$$\therefore M=\int_0^p (l+x)(p-x)dx \Big/ \int_0^p dx=\frac{1}{p}\int_0^p[lp+(p-l)x-x^2]dx$$
$$=\tfrac{1}{6}p(3l+p).$$

Also solved by G. B. M. Zerr.